A Belief-Based Account of Decision Under Uncertainty

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We develop a belief-based account of decision under uncertainty. This model predicts decisions under uncertainty from (i) judgments of probability, which are assumed to satisfy support theory; and (ii) decisions under risk, which are assumed to satisfy prospect theory. In two experiments, subjects evaluated uncertain prospects and assessed the probability of the respective events. Study 1 involved the 1995 professional basketball playoffs; Study 2 involved the movement of economic indicators in a simulated economy. The results of both studies are consistent with the belief-based account, but violate the partition inequality implied by the classical theory of decision under uncertainty.
(*Decision Making; Risk; Uncertainty; Expected Utility; Prospect Theory; Support Theory; Decision Weights; Judgment; Probability*)

1. Introduction

It seems obvious that the decisions to invest in the stock market, undergo a medical treatment, or settle out of court depend on the strength of people's beliefs that the market will go up, that the treatment will be successful, or that the court will decide in their favor. It is less obvious how to elicit and measure such beliefs. The classical theory of decision under uncertainty derives beliefs about the likelihood of uncertain events from people's choices between prospects whose consequences are contingent on these events. This approach, first advanced by Ramsey (1931),[1] gives rise to an elegant axiomatic theory that yields simultaneous measurement of utility and subjective probability, thereby bypassing the thorny problem of how to interpret direct expressions of belief.

From a psychological (descriptive) perspective, the classical theory can be questioned on several counts. First, it does not correspond to the common intuition that belief precedes preference. People typically choose to bet $50 on team *A* rather than team *B* because they believe that *A* is more likely to win; they do not infer this belief from the observation that the former bet is more attractive than the latter. Second and perhaps more important, the classical theory does not consider probability judgments that could be useful in explaining and predicting decisions under uncertainty. Third, and most important, the empirical evidence indicates that the major assumptions of the classical theory that underlie the derivation of belief from preference are not descriptively valid.

This article develops a belief-based account in which probability judgments are used to predict decisions under uncertainty. We first review recent work on probability judgment and on the weighting function of prospect theory that serves as the basis for the present development. We next formulate a two-stage model of decision under uncertainty, and explore its testable implications. This model is tested against the classical theory in two experiments. Finally, we address some empirical, methodological, theoretical, and practical issues raised by the present development.

2. Theoretical Background

There is an extensive body of research indicating that people's choices between risky prospects depart systematically

[1] The notion that beliefs can be measured based on preferences was anticipated by Borel (1924).

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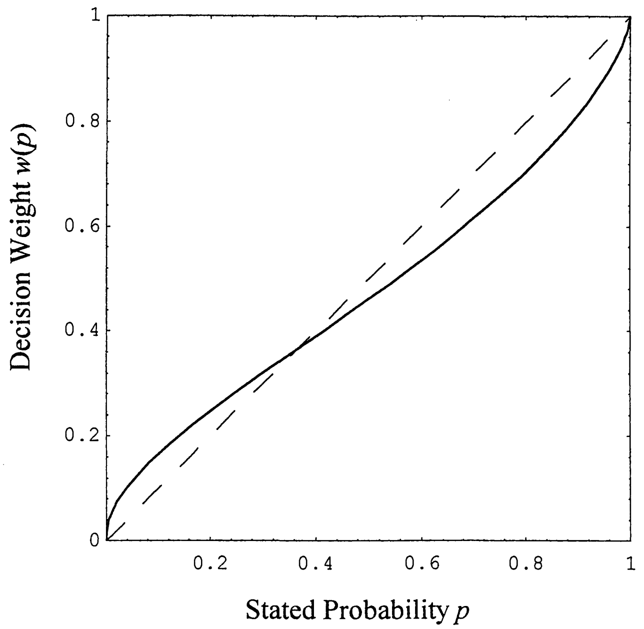
from expected utility theory (for a review, see Camerer 1995). Many of these violations can be explained by a nonlinear weighting function (see Figure 1) that overweights low probabilities and underweights moderate to high probabilities (Kahneman and Tversky 1979, Tversky and Kahneman 1992, Prelec 1998). Such a function accounts for violations of the independence axiom (the common consequence effect) and the substitution axiom (the common ratio effect), first demonstrated by Allais (1953). It also accommodates the commonly observed fourfold pattern of risk attitudes: risk seeking for gains and risk aversion for losses of low probability, together with risk aversion for gains and risk seeking for losses of high probability (Tversky and Kahneman 1992). Finally, it is consistent with the observed pattern of fanning in and fanning out in the probability triangle (Camerer and Ho 1994, Wu and Gonzalez 1998a).

Although most empirical studies have employed risky prospects, where probabilities are assumed to be known, virtually all real-world decisions (with the notable exception of games of chance) involve uncertain prospects (e.g., investments, litigation, insurance) where this assumption does not hold. In order to model such decisions we need to extend the key features of the

Figure 1 **Weighting Function for Decision under Risk,** $w(p) = \exp(-\beta(-\ln p)^{\alpha})$, **with** $\alpha = 0.7$, $\beta = 1$ **(Prelec 1998)**



risky weighting function to the domain of uncertainty. Tversky and Wakker (1995) established such a generalization, within the framework of cumulative prospect theory, by assuming that an event has more impact on choice when it turns an impossibility into a possibility or a possibility into a certainty than when it merely makes a possibility more or less likely.

Formally, let W denote the weighting function defined on subsets of a sample space S, where $W(\phi) = 0$ and $W(S) = 1$. W satisfies *bounded subadditivity* if:

(i) $W(A) \geq W(A \cup B) - W(B)$, and

(ii) $W(S) - W(S - A) \geq W(A \cup B) - W(B)$,

provided A and B are disjoint and $W(B)$ and $W(A \cup B)$ are bounded away from 0 and 1, respectively.[2] Condition (i) generalizes the notion that increasing the probability of winning a prize from 0 to p has more impact than increasing the probability of winning from q to $q + p$, provided $q + p < 1$. This condition reflects the *possibility effect*. Condition (ii) generalizes the notion that decreasing the probability of winning from 1 to $1 - p$ has more impact than decreasing the probability of winning from $q + p$ to q, provided $q > 0$. This condition reflects the *certainty effect*. Note that risk can be viewed as a special case of uncertainty where probability is defined via a standard chance device so that the probabilities of outcomes are known.

Tversky and Fox (1995) tested bounded subadditivity in a series of studies using both risky prospects and uncertain prospects whose outcomes were contingent on upcoming sporting events, future temperature in various cities, and changes in the Dow Jones index. The data satisfied bounded subadditivity for both risk and uncertainty. Furthermore, this effect was more pronounced for uncertainty than for risk, indicating greater departures from expected utility theory when probabilities are not known. The results of these experiments are consistent with a two-stage model in which the decision maker first assesses the probability P of an uncertain event A, then transforms this value using the risky weighting function,[3] w.

[2] The boundary conditions are needed to ensure that we always compare an interval that includes an endpoint to an interval that is bounded away from the other endpoint (see Tversky and Wakker 1995 for a more rigorous formulation).

[3] We use the lower case w to denote the weighting function for risk and the upper case W to denote the weighting function for uncertainty.

In the present article we elaborate this two-stage model and investigate its consequences. To simplify matters, we confine the present treatment to simple prospects of the form (x, A) that pay $\$x$ if the target event A obtains, and nothing otherwise.[4] We assume that the overall value V of such prospects is given by

$$V(x, A) = v(x)W(A) = v(x)w[P(A)], \qquad (1)$$

where v is the value function for monetary gains, w is the risky weighting function, and $P(A)$ is the judged probability of A. The key feature of this model, which distinguishes it from other theories of decision under uncertainty, is the inclusion of probability judgments. Note that if $W(A)$ can be expressed as $w[P(A)]$, as implied by Equation (1), we can predict decisions under uncertainty from decisions under risk and judgments of probability. We further assume that risky choices satisfy prospect theory[5] (Kahneman and Tversky 1979, Tversky and Kahneman 1992) and that judged probabilities satisfy support theory (Tversky and Koehler 1994, Rottenstreich and Tversky 1997), a psychological model of degree of belief to which we now turn.

There is ample evidence that people's intuitive probability judgments are often inconsistent with the laws of chance. In particular, different descriptions of the same event often give rise to systematically different responses (e.g., Fischhoff et al. 1978), and the judged probability of the union of disjoint events is generally smaller than the sum of judged probabilities of these events (e.g., Teigen 1974). To accommodate such findings, support theory assumes that (subjective) probability is not attached to events, as in other models, but rather to descriptions of events, called *hypotheses;* hence, two descriptions of the same event may be assigned different probabilities. Support theory assumes that each hypothesis A has a nonnegative support value $s(A)$ corresponding to the strength of the evidence for this hypothesis. The judged probability $P(A, B)$, that hypothesis A rather than B holds, assuming that one and only one of them obtains, is given by:

$$P(A, B) = \frac{s(A)}{s(A) + s(B)}, \qquad (2)$$

where

$$s(A) \leq s(A_1 \vee A_2) \leq s(A_1) + s(A_2), \qquad (3)$$

provided (A_1, A_2) is recognized as a partition of A.

In this theory, judged probability is interpreted as the support of the focal hypothesis A relative to the alternative hypothesis B (equation 2). The theory further assumes that (i) unpacking a description of an event A (e.g., homicide) into disjoint components $A_1 \vee A_2$ (e.g., homicide by an acquaintance, A_1, or homicide by a stranger, A_2) generally increases its support, and (ii) the sum of the support of the component hypotheses is at least as large as the support of their disjunction (Equation (3)). The rationale for these assumptions is that (i) unpacking may remind people of possibilities that they have overlooked, and (ii) the separate evaluation of hypotheses tends to increase their salience and enhance their support.

Equation (2) implies *binary complementarity*: $P(A, B) + P(B, A) = 1$. For finer partitions, however, Equations (2) and (3) imply *subadditivity*: the judged probability of A is less than or equal to the sum of judged probabilities of its disjoint components. These predictions have been confirmed in several studies reviewed by Tversky and Koehler (1994). For example, experienced physicians were provided with medical data regarding the condition of a particular patient who was admitted to the emergency ward, and asked to evaluate the probabilities of four mutually exclusive and exhaustive prognoses. The judged probability of a prognosis (e.g., that the patient will survive the hospitalization) against its complement, evaluated by different groups of physicians, summed to one, in accord with binary complementarity. However, the sum of the judged probabilities for the four prognoses was substantially greater than one, in accord with subadditivity (Redelmeier et al. 1995).

Implications

Perhaps the most striking contrast between the two-stage model and the classical theory (i.e., expected utility theory with risk aversion) concerns the effect of partitioning. Suppose $(A_1, \ldots, A_n)$ is a partition of A, and $C(x, A)$ is the certainty equivalent of the prospect that

[4] The two-stage model has not yet been extensively tested for multiple nonzero outcomes; however, see Wu and Gonzalez (1998c) for a preliminary investigation.

[5] For the simple prospects considered here, the separable and cumulative versions of the theory are identical.

pays \$$x$ if A occurs, and nothing otherwise. The classical theory implies the following *partition inequality*:

$$C(x, A_1) + \cdots + C(x, A_n) \leq C(x, A), \qquad (4)$$

for all real x and $A \subset S$. That is, the certainty equivalent of an uncertain prospect exceeds the sum of certainty equivalents of the subprospects (evaluated independently) obtained by partitioning the target event. In the context of expected utility theory, the partition inequality is implied by risk aversion.[6] However, if people follow the two-stage model, defined in Equation (1), and if the judged probabilities are subadditive, as implied by support theory, then the partition inequality is not expected to hold. Such failures are especially likely when the curvature of the value function (between 0 and \$$x$) is not very pronounced and the target event (A) is partitioned into many components. Thus, the partition inequality provides a simple method for testing the classical theory and contrasting it to the two-stage model.

To test the two-stage model, we predict the certainty equivalent of an uncertain prospect, $C(x, A)$, from two independent responses: the judged probability of the target event, $P(A)$, and the certainty equivalent of the risky prospect, $C(x, P(A))$. It follows readily from Equation (1) that

$$\text{if } P(A) = p, \quad \text{then } C(x, A) = C(x, p). \qquad (5)$$

This condition provides a method for testing the two-stage model that does not require an estimation of the value function. The following two studies test the partition inequality and compare the predictions derived from Equation (5) to those of the classical theory.

3. Experiments

Study 1: Basketball Playoffs
Method

Participants. The participants in this study were 50 students at Northwestern University (46 men, 4 women; median age = 20) who responded to fliers calling for fans of professional basketball to take part in a study of decision making. Subjects indicated that they had watched several games of the National Basketball Association (NBA) during the regular season (median = 25). They received \$10 for completing a one-hour session and were told that some participants would be selected at random to play one of their choices for real money and that they could win up to \$160.

Procedure. The experiment was run using a computer. All subjects were run on the same day, during the beginning of the NBA quarterfinals. Subjects were given detailed instructions and an opportunity for supervised practice. The study consisted of four tasks.

The first task was designed to estimate subjects' certainty equivalents (abbreviated C) for risky prospects. These prospects were described in terms of a random draw of a single poker chip from an urn containing 100 chips numbered consecutively from 1 to 100. Nineteen prospects of the form (\$160, p) were constructed where p varied from 0.05 to 0.95 in multiples of .05. For example, the (\$160, .25) prospect would pay \$160 if the number of the poker chip is between 1 and 25, and nothing otherwise.

Each trial involved a series of choices between a prospect and an ascending series of sure payments (e.g., receive \$40 for sure). The order of the 19 risky prospects was randomized separately for each subject. Certainty equivalents were inferred from two rounds of such choices. The first round consisted of nine choices between the prospect and sure payments that were spaced evenly from \$0 to \$160. After completing the first round of choices, a new set of nine sure payments was presented, spanning the narrower range between the lowest payment that the subject had accepted and the highest payment that the subject had rejected (excluding the endpoints). The program enforced dominance and internal consistency within a given trial. For example, the program did not allow a respondent to prefer \$30 over a prospect and also prefer the same prospect over \$40. The program allowed subjects to backtrack if they felt they had made a mistake in the previous round of choices.

For each risky prospect, C was determined by linear interpolation between the lowest value accepted and the

[6] To demonstrate, set $u(0) = 0$. Hence, $C(x, A_1) + \cdots + C(x, A_n) = u^{-1}(u(x)\mathcal{P}(A_1)) + \cdots + u^{-1}(u(x)\mathcal{P}(A_n)) \leq u^{-1}(u(x)\mathcal{P}(A)) = C(x, A)$ if u is concave. We use $\mathcal{P}$ to denote an additive probability measure, to be distinguished from P, that denotes judged probability.

highest value rejected in the second round of choices, yielding a margin of error of ± \$1.00. Note that although our analysis is based on C, the data consisted of a series of choices between a given prospect and sure outcomes. Thus, respondents were not asked to generate C; it was inferred from their choices.

The second task was designed to estimate certainty equivalents for uncertain prospects. Each prospect offered to pay \$160 if a particular team, division, or conference would win the 1995 NBA championship. At the time of the study, eight teams remained (Chicago, Indiana, Orlando, New York, Los Angeles, Phoenix, San Antonio, Houston) representing four divisions (Central, Atlantic, Pacific, Midwestern) and two conferences (Eastern, Western). Fourteen prospects of the form (\$160, A) were constructed that offered to pay \$160 if a particular team, division, or conference were to win the 1995 NBA championship. For example, a typical prospect would pay \$160 if the Chicago Bulls win the championship. The elicitation method was identical to that of the first task.

The third task was designed to provide an independent test of risk aversion that makes no assumptions regarding the additivity of subjective probabilities or decision weights. Subjects were presented with a "fixed" prospect of the form (\$$a$, 0.25; \$$b$, 0.25; \$0, 0.50) and a "variable" prospect of the form (\$$c$, 0.25; \$$x$, 0.25; \$0, 0.50). These prospects were displayed as "spinner games" that would pay the designated amount depending on the particular region on which the spinner would land. In each trial, the values of a, b, and c were fixed, while the value of x varied. The initial value of x was set equal to b. Eight such pairs of prospects were constructed (see Table 1), presented in an order that was randomized separately for each subject. On each trial, participants were asked to indicate their preference between the prospects. When a subject preferred the fixed prospect, the value of x increased by \$16; when a subject preferred the variable prospect, the value of x decreased by \$16. When a subject's preference switched from the fixed prospect to the variable prospect or from variable to fixed, the change in x reversed direction and the increment was cut in half (i.e., from \$16 to \$8, from \$8 to \$4, and so forth) until the increment was \$1. This process was repeated until the subject indicated that the two prospects were equally attractive. The program did not allow subjects to violate dominance.

Table 1 **Values of *a*, *b*, and *c* Used in the Spinner Games of Study 1 and Median Value of Subjects' Responses (*x*)**

	Fixed Prospect			Variable Prospect		
Probability	0.25	0.25	0.50	0.25	0.25	0.50
Outcome	\$$a$	\$$b$	\$0	\$$c$	\$$x$ (Median)	\$0
1)	50	100		25	131	
2)	30	60		10	86.5	
3)	20	90		40	70	
4)	10	110		35	82	
5)	85	55		120	31	
6)	50	45		75	29	
7)	95	25		70	42	
8)	115	15		80	43	

The fourth task required participants to estimate the probability of each target event (i.e., that a particular team, division, or conference would win the NBA playoffs). The fourteen events were presented in an order that was randomized separately for each subject. On each trial, subjects could respond by either typing a number between 0 and 100, or by clicking and dragging a "slider" on a visual scale.

Subjects performed two additional tasks. They judged the probability that one team rather than another would win the NBA championship assuming that two particular teams reach the finals, and they rated the "strength" of each team. These data are discussed in Fox (1998).

Results

Judged Probability. The median judged probability for each target event is listed in Figure 2. The figure shows that the sum of these probabilities is close to one for the two conferences, nearly one and a half for the four divisions, and more than two for the eight teams. This pattern is consistent with the predictions of support theory that

$$\sum_{\text{teams}} P \geq \sum_{\text{divisions}} P \geq \sum_{\text{conferences}} P , \quad (6)$$

and the sum over the two conferences equals one. Moreover, in every case the sum of the probabilities for the individual teams is greater than the probability of the respective division, and the sum of the probabilities for

Figure 2 Median Judged Probabilities for All Target Events in Study 1

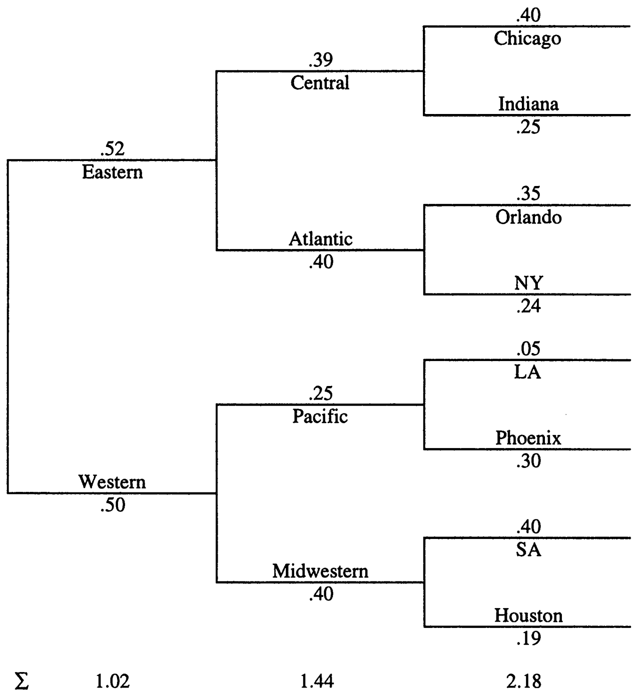
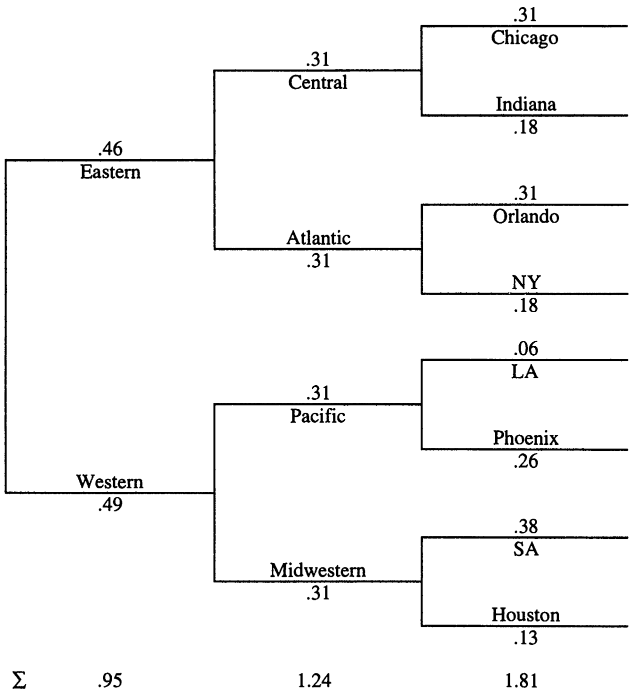


Figure 3 Median Normalized Certainty Equivalents for All Prospects in Study 1



the divisions is greater than the probability of the respective conference, consistent with support theory.[7]

The same pattern holds in the analysis of individual subjects. The median sum of probabilities for the eight teams was 2.40, the median sum for the four divisions was 1.44, and the median sum of probabilities for the two conferences was 1.00. Moreover, 41 of 50 respondents satisfied Equation (6) with strict inequalities, and 49 of 50 respondents reported probabilities for the eight teams that summed to more than one ($p < 0.001$ by sign test in both cases).

Certainty Equivalents. Figure 3 presents the median normalized C for each prospect; that is, the median certainty equivalent divided by \$160. The choice data in Figure 3 echo the judgment data in Figure 2. In every case, the sum of Cs for the individual teams is greater than C for the respective division, and the sum of Cs for the divisions is greater than C for the respective conference.[8] Furthermore, the sum of Cs for the 8 teams exceeds \$160; that is, the sum of the normalized Cs is greater than one.

Again, the same pattern holds in the analysis of individual subjects. The median sum of normalized Cs for the 8 teams was 2.08, the median sum for the 4 divisions was 1.38, and the median sum for the 2 conferences was 0.93. Moreover, the pattern implied by the partition inequality (Equation (4)):

$$\sum_{\text{teams}} C \leq \sum_{\text{divisions}} C \leq \sum_{\text{conferences}} C,$$

was satisfied by only one respondent, whereas 41 of the 50 respondents satisfied the reverse pattern that is consistent with the two-stage model ($p < 0.001$):

$$\sum_{\text{teams}} C > \sum_{\text{divisions}} C > \sum_{\text{conferences}} C.$$

[7] In every case this also holds for a significant majority of subjects ($p < 0.01$ by sign tests).

[8] In every case this also holds for a significant majority of subjects ($p < 0.01$).

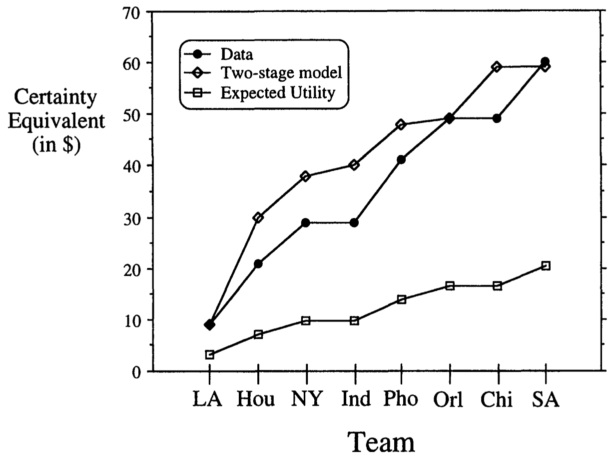
Furthermore, only 5 subjects produced Cs for the 8 teams that summed to less than \$160, whereas 44 subjects produced Cs that summed to more than \$160 ($p < 0.001$). This pattern violates the partition inequality, with $A = S$.

Comparing Models. We next compare the fit of the classical theory to that of the two-stage model. For each event A, we observed the median judged probability $P(A)$, then searched for the median C of the risky prospect (x, p) where $p = P(A)$. For example, the median judged probability that the San Antonio Spurs (SAS) would win the NBA championship was 0.40, and the median value of C(\$160, .40), was \$59. According to Equation (5), therefore, C(\$160, SAS) should equal \$59; the actual value was \$60. In cases where the $P(A)$ is not a multiple of 5 percent, we determined the certainty equivalent by linear interpolation.

To fit the classical theory, let C_A be the certainty equivalent of the prospect (\$160, A). Setting $u(0) = 0$, the classical theory yields $u(C_A) = u(160)\mathcal{P}(A)$, where u is concave and $\mathcal{P}(A)$ is an additive (subjective) probability measure. Hence, $\mathcal{P}(A) = u(C_A)/u(160)$. Previous studies (e.g., Tversky 1967, Tversky and Kahneman 1992) have indicated that the value function for small to moderate gains can be approximated by a power function of the form $u(x) = x^\alpha$, $\alpha > 0$. To estimate the exponent, we used data from the "spinner games" described above. If a subject is indifferent between the fixed prospect (\$$a$, 0.25; \$$b$, 0.25; \$0, 0.5) and the variable prospect (\$$c$, 0.25; \$$x$, 0.25; \$0, 0.5) then assuming a power utility function, $a^\alpha + b^\alpha = c^\alpha + x^\alpha$. Because a, b, and c are given and the value of x is determined by the subject, one can solve for $\alpha > 0$. The exponent for each subject was estimated using the median value of α over the eight problems listed in Table 1. This analysis showed that participants were generally risk-averse: 32 subjects exhibited $\alpha < 1.00$ (risk aversion); 14 exhibited $\alpha = 1.00$ (risk neutrality); and 4 exhibited $\alpha > 1.00$ (risk seeking) ($p < 0.001$ by sign test). The median response to each of the eight trials yielded $\alpha = 0.80$. The finding that the majority of subjects exhibited risk aversion in this task shows that the violations of the partition inequality described earlier cannot be explained by a convex utility function.

Subjective probabilities were estimated as follows. For each elementary target event A, we computed $(C_A/160)^\alpha$ and divided these values by their sum to ensure additivity. Figure 4 displays the median C for each of the eight teams along with the predictions of the two-stage model and the standard theory (assuming $\alpha = 0.80$, based on the median response to each item). It is evident from the figure that the two-stage model fits the data (mean absolute error = \$5.83) substantially better than does the standard theory (mean absolute error = \$23.71).[9] The same pattern is evident in the responses of individual subjects. The two-stage model fits the data better than does the classical theory for 45 of the 50 subjects ($p < 0.001$).

Figure 4 **Median Certainty Equivalents of Bets for All Eight Teams, and Predictions of Two-Stage Model and Classical Theory (with α = 0.80)**



Note that the predictions of the two-stage model were derived from two independent tasks; no parameters were estimated from the fitted data. In contrast, the predictions of the classical theory were derived by estimating a parameter for each of the fitted data points; these estimates were constrained only by the requirement that the subjective probabilities sum to unity. In light of the substantial advantage conferred to the classical theory in this comparison, its inferior fit provides compelling evidence against the additivity of subjective probabilities that are inferred from choice.

Studies of Unpacking. We have attributed the failure of the partition inequality to the subadditivity of judged

[9] A more conservative test of the standard theory assuming $\alpha = 1.00$ yields a mean absolute error of \$13.29.

probability that is implied by support theory. A more radical departure from the classical theory is suggested by the *unpacking principle* of support theory, according to which unpacking the description of an event into an explicit disjunction of constituent events generally increases its judged probability. Under the two-stage model, therefore, unpacking the description of an event is also expected to increase the attractiveness of a prospect whose outcome depends on this event. Furthermore, if this effect is sufficiently pronounced, it can give rise to violations of monotonicity where $C(x, A) < C(x, A_1 \vee \cdots \vee A_n)$ even when $A_1 \vee \cdots \vee A_n$ is a proper subset of A.

To explore this possibility, we presented a brief questionnaire to 58 business students at Northwestern University shortly before the beginning of the 1996 NBA playoffs. The survey was administered in a classroom setting. Prior to the survey, respondents were presented with the records of all NBA teams listed by their division and conference. Subjects were randomly assigned to one of two groups. Subjects in the first group ($N = 28$) stated their certainty equivalent for two prospects: a prospect that offered $75 if the winner of the 1996 playoffs belongs to the Eastern conference, and a prospect that offered $75 if one of the four leading Western conference teams (Seattle, Utah, San Antonio, or Los Angeles) would win the 1996 playoffs. Subjects in the second group ($N = 30$) stated their certainty equivalent for the two parallel prospects: a prospect that offered $75 if the winner of the 1996 playoffs belongs to the Western conference, and a prospect that offered $75 if one of the four leading Eastern conference teams (Chicago, Orlando, Indiana, or New York) would win the 1996 playoffs.[10] Each group also assessed the probability of the two target events that defined the prospects evaluated by the other group. For example, the group that evaluated the prospect that would pay if an Eastern team will win assessed the probability that a Western team will win, and vice versa.

Table 2 presents the median judged probability and certainty equivalent for the two conferences, and the four leading teams in each conference. Although these teams had the best record in their respective conferences, some strong teams (e.g., the defending champion Houston Rockets) were not included in the list. Monotonicity requires, therefore, that the judged probability and certainty equivalent assigned to each conference should exceed those assigned to their leading teams. The unpacking principle, on the other hand, suggests that a nontransparent comparison (e.g., a between-subjects test) may produce violations of monotonicity. Indeed, the data of Table 2 do not satisfy the monotonicity requirement. There is essentially no difference in either judged probability or the certainty equivalent between the Western conference and its four leading teams, whereas the judged probability and the certainty equivalent assigned to the Eastern conference are significantly smaller than those assigned to its four leading teams ($p < 0.05$, by a t-test in each case).[11]

Table 2 **Median Judged Probability and Certainty Equivalent for the Two Conferences and Respective Leading Teams for the 1996 NBA Playoffs**

	Judged Probability	Certainty Equivalents
Eastern Conference	0.78	$50
Chi ∨ Orl ∨ Ind ∨ NY	0.90	$60
Western Conference	0.18	$15
Seattle ∨ Utah ∨ SA ∨ LA	0.20	$15

Violations of monotonicity (or dominance) induced by unpacking have been observed by several investigators. Johnson et al. (1993), for example, reported that subjects were willing to pay more for a health insurance policy that covers hospitalization for all diseases and accidents than for a policy that covers hospitalization for any reason. Wu and Gonzalez (1998b) found similar effects in the evaluation of prospects contingent on diverse events such as the winner of the World Series, the outcomes of the 1996 elections, and future temperature in Boston. Although the effects observed in the above studies are not very pronounced, they indicate that unpacking can give rise to nonmonotonicity in judgments

[10] Eight teams qualified for the playoffs from each conference.

[11] Violations of monotonicity are also evident in the certainty equivalent data for Study 1 reported in Figure 3. Note that the median certainty equivalent for San Antonio is higher than the median certainty equivalent for the Midwestern Division; Chicago and Orlando are priced as high as their respective divisions. While these results are consistent with the present account, none of these differences is statistically significant.

of probability as well as the pricing of uncertain prospects.

Study 2: Economic Indicators

The above study, like previous tests of bounded subadditivity, relied on subjects' beliefs regarding the occurrence of various real-world events. In the following study, subjects were given an opportunity to learn the probability of target events by observing changes in inflation and interest rates in a simulated economy. This design allows us to test both the classical theory and the two-stage model in a controlled environment in which all subjects are exposed to identical information. It also allows us to compare subjects' judged probabilities to the actual probabilities of the target events.

Method

Participants. Subjects were students ($N = 92$) enrolled in an introductory class in judgment and decision making at Stanford University. Students were asked to download a computer program from a world wide web page, run the program, and e-mail their output to a class account. At the time of the study, the students had been exposed to discussions of probability theory and judgmental biases, but they were unfamiliar with decision theory. We received 86 complete responses. Four subjects were dropped because they apparently did not understand the instructions. The 82 remaining subjects included 49 men and 33 women (median age = 21.5). Most of them completed the study in less than an hour (median = 47 minutes).

Procedure. Subjects were first given an opportunity to learn the movement of two indicators (inflation and interest rates) in a simulated economy. Each indicator could move either up or down relative to the previous quarter. In this economy both indicators went up 60 percent of the time, inflation went up and interest went down 25 percent of the time, inflation went down and interest went up 10 percent of the time, and both indicators went down 5 percent of the time. The order of these events was randomized over 60 quarters of learning, separately for each subject. Participants were informed that the probabilities of the target events were the same for each quarter.

The learning procedure was divided into two parts. During the first 20 quarters, subjects merely clicked the mouse to advance to the next quarter and observed what happened. During the remaining 40 quarters, subjects also played a game in which they predicted the direction that each indicator would move in the subsequent quarter, and they made (hypothetical) bets on their predictions. After each prediction, subjects were given feedback and the computer adjusted their "bank balance" according to whether they had predicted correctly.

The second task was designed to estimate C for risky prospects. We constructed eleven prospects of the form (\$1600, p) that offered to pay \$1600 with probability (0.01, 0.05, 0.10, 0.15, 0.25, 0.50, 0.75, 0.85, 0.90, 0.95, 0.99). The elicitation procedure was identical to that used in the basketball study, except that all dollar amounts were multiplied by 10. Using this method we could estimate C for \$1600 prospects within ± \$10.

The third task was designed to estimate C for uncertain prospects. Subjects were first given an opportunity to review up to three times a 35-second "film" that very briefly displayed changes in the two indicators over each of the 60 quarters that subjects had previously observed. They were then presented with prospects that offered \$1600 contingent on the movement of the indicators in the next (i.e., 61st) quarter. The first four trials involved movement of a single indicator (e.g., win \$1600 if inflation up). The next four trials involved movement of both indicators (e.g., win \$1600 if inflation up and interest down). The final four trials involved negations of the previous four events (e.g., win \$1600 *unless* inflation up and interest down). The order of prospects within each set of trials was randomized separately for each subject. C was elicited through a series of choices between uncertain prospects and sure payments, as in the previous task.

The fourth task was designed to obtain an independent test of risk aversion. The procedure was essentially identical to the third task of the basketball study, except that the dollar amounts were multiplied by 10, and the initial value of x for the variable prospect was set so that the expected value of the two spinner games was equal (see Table 7).

In the fifth task, subjects judged the probability of each target event. Subjects were first given an opportunity to review again up to three times a "film" of the 60 quarters they had previously observed. The first eight trials involved the movement of a single economic indicator (e.g., what is the probability that the following happens: inflation up) or combination of indicators (e.g., inflation up and interest down). The last four trials involved complementary events (e.g., what is the probability that the following does *not* happen: inflation up and interest down). The order of these events within each set was randomized separately for each subject, and responses were elicited as in the basketball study.

Subjects performed one additional task involving the acceptability of mixed prospects. The results of this task will not be discussed here.

Results

Judged Probabilities. Figure 5 plots for each target event the median judged probability against the actual probability. The figure shows that participants had learned the probabilities of the target events with impressive accuracy ($r = 0.995$). The mean absolute difference (MAD) between median judged probability and actual probability was 0.048. The median correlation for individual subjects was 0.89 (median MAD = 0.14). Subjects also exhibited a tendency to overestimate low probabilities and underestimate high probabilities. Of the 82 subjects, 60 both overestimated, on average, events with true probabilities less than 50 percent, and underestimated, on average, events with true probabilities greater than 50 percent ($p < 0.001$ by sign test).

The median judged probability for each target event is listed in Table 3. Each cell displays the probability that the two indicators move as specified. The median judged probabilities of the complementary events are given in brackets. For example, the median judged probability that both indicators go up is 0.60, the probability that it is not the case that both indicators go down is 0.85, and the probability that inflation goes up is 0.75.

Recall that support theory predicts that the judged probability of an event and its complement will sum to unity (binary complementarity), but in all other cases the sum of the judged probabilities of disjoint events

Figure 5 Median Judged Probability as a Function of Actual Probabilities for All Target Events in Study 2

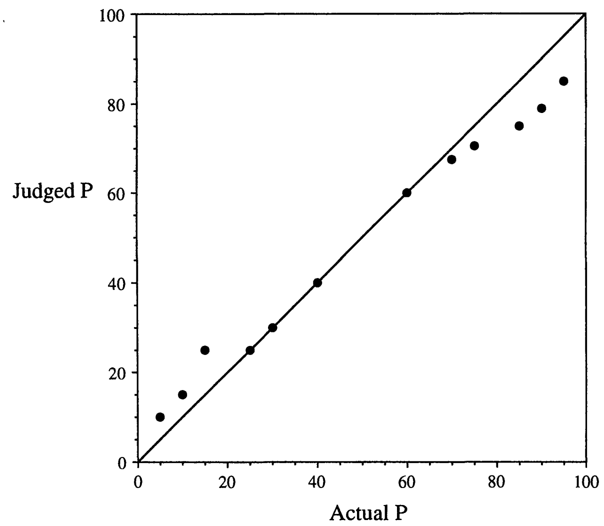


Table 3 Median Judged Probability of All Target Events in Study 2 (Data for Complementary Events are Given in Brackets)

		Interest: Up	Interest: Down	
Inflation	Up	0.60 [0.40]	0.25 [0.71]	0.75
	Down	0.15 [0.79]	0.10 [0.85]	0.25
		0.68	0.30	

Table 4a Tests of Binary Complementarity for Median Judged Probabilities in Study 2

Partition	ΣP_i	$\Delta = \Sigma P_i - 1$
$U\bullet$, $D\bullet$	1.00	0.00
$\bullet U$, $\bullet D$	0.98	−0.02
UU, $\overline{UU}$	1.00	0.00
UD, $\overline{UD}$	0.96	−0.04
DU, $\overline{DU}$	0.94	−0.06
DD, $\overline{DD}$	0.95	−0.05
Mean	0.97	−0.03

The first column presents binary partitions of S, the second column (ΣP_i) presents the sum of median judged probabilities for this partition, and the third column (Δ) presents the difference between this sum and one.

will be greater than or equal to the judged probability of their union (subadditivity). Table 4a presents six tests of binary complementarity, based on the median response to each item. Each row presents a binary partition of the sample space, along with the sum of median judged probabilities for this partition. The column labeled Δ lists the difference between this sum and one. For each event, the first letter corresponds to inflation (*U* for up, *D* for down) and the second to interest. For example, *UD* is the event "inflation up and interest down," *U*• is the event "inflation up," and •*D* is the event "interest down." Complements are denoted by a bar. For example, $\overline{UU}$ is the event "it is not the case that interest up and inflation up." As expected, the sum of median judged probabilities for complementary events is close to unity (mean = 0.97). However, these values were systematically smaller than one: the median value of the mean of these tests for each subject is 0.98; 48 subjects exhibited a mean less than 1.00, 10 exhibited a mean equal to 1.00, and 24 exhibited a mean greater than 1.00 ($p < 0.01$).

Table 4b presents eight tests of subadditivity based on median judged probabilities. Each row presents the sum of judged probabilities of disjoint events, the judged probability of their union, and the difference between them (Δ). For example, the first row shows that *P*(*inflation Up and interest Up*) + *P*(*inflation Up and interest Down*) = 0.85, and *P*(*inflation Up*) = 0.75, so that Δ = 0.10. As expected, Table 4b shows that in every case the sum of judged probabilities of disjoint events is greater than or equal to the judged probability of their union,[12] and the mean difference between them is 0.10. Furthermore, 60 of 82 subjects exhibited this pattern (i.e., mean $\Delta > 0$) on average ($p < 0.001$ by sign test).

Table 4b **Tests of Subadditivity for Median Judged Probabilities in Study 2**

Event	Partition	ΣP_i	P	$\Delta = \Sigma P_i - P$
U•	*UU*, *UD*	0.85	0.75	0.10
D•	*DU*, *DD*	0.25	0.25	0.00
•*U*	*UU*, *DU*	0.75	0.68	0.07
•*D*	*UD*, *DD*	0.35	0.30	0.05
$\overline{UU}$	*UD*, *DU*, *DD*	0.50	0.40	0.10
$\overline{UD}$	*UU*, *DU*, *DD*	0.85	0.71	0.14
$\overline{DU}$	*UU*, *UD*, *DD*	0.95	0.79	0.16
$\overline{DD}$	*UU*, *UD*, *DU*	1.00	0.85	0.15
Mean		0.69	0.59	0.10

The first column presents a target event, the second column presents a partition of that event, the third column (ΣP_i) presents the sum of median judged probabilities over the partition, the fourth column (*P*) presents the median judged probability of the target event, and the fifth column (Δ) presents the difference between these two values.

Certainty Equivalents. The median normalized C_A for each target event *A* is presented in Table 5. The corresponding medians for the complementary events are given in brackets. For example, the median normalized *C* for the event that both indicators go up is 0.43 and the median for the complementary event is 0.29. It can be shown that whenever $w(p) + w(1 - p) \leq 1$ and the value function is concave, the two-stage model implies the partition inequality for binary partitions of the sample space[13] (i.e., $C(x, A) + C(x, S - A) \leq C(x, S) = x$), but it does not imply the partition inequality for finer partitions of *S* or for binary partitions of other events.

Table 6a presents six tests of the partition inequality for binary partitions of the sample space. Analogous to Table 4a, each row presents a binary partition of *S* along with the sum of median normalized *C*s for this partition and the difference (Δ) between this sum and one. As predicted by both the classical theory and the present account, the partition inequality holds for all comparisons listed in Table 6a (mean $\Delta = -0.24$). It also holds on average for 68 of 82 subjects ($p < 0.001$).

Table 6b presents eight additional tests of the partition inequality based on proper subsets of *S*. Analogous to Table 4b, each entry presents the sum of median normalized *C*s of disjoint events, the normalized *C* of their union, and the difference between them. Table 6b shows that the partition inequality fails in all cases (mean Δ = 0.09).[14] Furthermore, 51 of 82 subjects exhibited this pattern on average (i.e., $\Delta > 0$, $p < 0.05$ by sign test).

[12] In every case $\Delta > 0$ for a significant majority of subjects ($p < 0.05$).

[13] The condition $w(p) + w(1 - p) \leq 1$, called *subcertainty*, is generally supported by empirical data (see e.g., Tversky and Kahneman 1992). It says that the certainty effect is more pronounced than the possibility effect, and it implies the common finding that $w(0.5) < 0.5$ (see Figure 1).

[14] In every case $\Delta > 0$ for a majority of subjects; this majority is statistically significant ($p < 0.05$ by sign test) for all tests but the first and fourth listed in the table.

Table 5 Median Normalized Certainty Equivalents of All Target Events in Study 2 (Data for Complementary Events are Given in Brackets)

		Interest		
		Up	Down	
Inflation	Up	0.43 [0.29]	0.22 [0.48]	0.62
	Down	0.13 [0.59]	0.10 [0.76]	0.18
		0.49	0.28	

The preceding results can be summarized as follows. For binary partitions of the sample space S, judged probabilities (nearly) satisfy binary complementarity (Table 4a), and certainty equivalents satisfy the partition inequality (Table 6a). This pattern is consistent with both the classical theory and the present account. For finer partitions, however, the data yield subadditivity for judged probabilities (Table 4b) and reversal of the partition inequality for certainty equivalents (Table 6b). This pattern is consistent with the two-stage model but not with the classical theory.

Comparing Models. We next compare the fit of the classical theory to that of the two-stage model using the same method as in the previous study. To fit the classical theory, the exponent α of the utility function was estimated from the spinner games and the exponent for each subject was estimated using the median value of α derived from that subject's responses to the eight problems listed in Table 7. Subjects were generally risk-averse: 48 subjects exhibited $\alpha < 1.00$ (risk aversion); 32 exhibited $\alpha = 1.00$ (risk neutrality); and 2 exhibited $\alpha > 1.00$ (risk seeking) ($p < 0.001$). Applying the same analysis to the median response to each of the eight trials yields $\alpha = 0.80$.

Table 6a Tests of the Partition Inequality for Binary Partitions of *S* in Study 2

Partition	ΣC_i	$\Delta = \Sigma C_i - 1$
$U\bullet$, $D\bullet$	0.79	−0.21
$\bullet U$, $\bullet D$	0.78	−0.22
UU, $\overline{UU}$	0.73	−0.27
UD, $\overline{UD}$	0.70	−0.30
DU, $\overline{DU}$	0.72	−0.28
DD, $\overline{DD}$	0.86	−0.14
Mean	0.76	−0.24

The first column presents a partition of S, the second column (ΣC_i) presents the sum of median normalized certainty equivalents for this partition, and the third column (Δ) presents the difference between this sum and one.

Table 6b Tests of the Partition Inequality in Study 2 for Proper Subsets of *S*

Event	Partition	ΣC_i	C	$\Delta = \Sigma C_i - C$
$U\bullet$	UU, UD	0.65	0.62	0.03
$D\bullet$	DU, DD	0.23	0.18	0.05
$\bullet U$	UU, DU	0.56	0.49	0.07
$\bullet D$	UD, DD	0.32	0.28	0.04
$\overline{UU}$	UD, DU, DD	0.45	0.29	0.16
$\overline{UD}$	UU, DU, DD	0.66	0.48	0.18
$\overline{DU}$	UU, UD, DD	0.75	0.59	0.16
$\overline{DD}$	UU, UD, DU	0.78	0.76	0.02
Mean		0.55	0.46	0.09

The first column presents a target event, the second column presents a partition of that event, the third column (ΣC_i) presents the sum of median normalized certainty equivalents over the partition, the fourth column (C) presents the median normalized certainty equivalent of the target event, and the fifth column (Δ) presents the difference between these two values.

Table 7 Values of *a*, *b*, and *c* Used in Spinner Games of Study 2, and Median Value of Subjects' Responses (*x*)

	Fixed Prospect			Variable Prospect		
Probability	0.25	0.25	0.50	0.25	0.25	0.50
Outcome	\$*a*	\$*b*	\$0	\$*c*	\$*x* (median)	\$0
1)	500	1000		250	1330	
2)	500	700		250	990	
3)	200	1200		400	985	
4)	200	800		400	600	
5)	650	550		800	400	
6)	650	350		800	210	
7)	1100	100		750	360	
8)	1100	250		750	520	

According to the classical theory with a power utility function, $C_A^\alpha = 1600^\alpha \mathcal{P}(A)$. Recall that in this study subjects learned probabilities by observing the frequencies of the four elementary events (e.g., inflation up and interest up). For each elementary target event A, we computed $(C_A/1600)^\alpha$, and divided these values by their sum to ensure additivity. The subjective probabilities of all other events were derived from these estimates, assuming additivity.

The two-stage model was estimated using Equation (5) as in the previous study. The data show that this model fits the median certainty equivalents (mean absolute error = \$69) better than the classical theory (mean absolute error = \$128).[15] The same holds within the data of individual subjects. Using individual estimates of the parameters, the two-stage model fits the data better than the standard theory for 50 of the 82 participants ($p < 0.05$).

4. Discussion

The two preceding studies indicate that to a reasonable first approximation, the certainty equivalents of uncertain prospects can be predicted from independent judgments of probability and certainty equivalents for risky prospects, without estimating any parameters from the fitted data. Moreover, this model can account for the observed violations of the partition inequality. We conclude this article with a review of related studies, a comment regarding response bias, a discussion of the problem of source preference, and some closing thoughts concerning practical implications of the two-stage model.

Previous Studies

In the basketball study reported above, the event space has a hierarchical structure (conferences, divisions, teams). In the economic indicators study, the event space has a product structure (inflation up/down × interest up/down). Previous tests of bounded subadditivity employed a dimensional structure in which a numerical variable (e.g., the closing price per share of Microsoft stock two weeks in the future) was partitioned into intervals (e.g., less than \$88, \$88 to \$94, more than \$94). Subjects priced prospects contingent on these events and assessed their probabilities.

The results of these studies, summarized in Table 8, are consistent with the present account. First, consider probability judgments. The column labeled $(A, S - A)$ presents the median sum of judged probabilities for binary partitions of S, and the column labeled $(A_1, \ldots, A_n)$ presents the median sum of judged probabilities for finer partitions of S. The results conform to support theory: sums for binary partitions of S are close to one, whereas sums for n-fold partitions are consistently greater than one. Next, consider certainty equivalents. The column labeled ΣC presents the median sum of normalized certainty equivalents for the finest partition of S in each study, and the column labeled $\%V$ presents the corresponding percentage of subjects who violated the partition inequality. In accord with the present findings, the majority of subjects in every study violated the partition inequality, and the sum of certainty equivalents was often substantially greater than the prize. This pattern holds for a wide range of sources, with and without monetary incentives, and for both naive and expert subjects. Taken together, these findings suggest that subadditivity of judged probability is a major cause of violations of the partition inequality.

The studies of Fox et al. (1996) are particularly interesting in this respect. Participants were professional options traders who priced prospects contingent on the closing price of various stocks. Unlike typical subjects, the options traders priced risky prospects by their expected value, yielding $v(x) = x$ and $w(p) = p$. Like most other subjects, however, their judged probabilities were subadditive (i.e., $P(A_1) + \cdots + P(A_n) > P(A)$). Under these circumstances, the two-stage model predicts

$$C(x, A_1) + \cdots + C(x, A_n) = P(A_1)x + \cdots + P(A_n)x > P(A)x = C(x, A),$$

whereas the classical theory requires equality throughout. The data for the options traders, summarized in Table 8, confirms the prediction of the two-stage model.

Response Bias

We have attributed the subadditivity of judged probabilities and of decision weights to basic psychological

[15] A least-square procedure for estimating all subjective probabilities simultaneously subject to the additivity constraint did not improve the fit of the classical theory.

Table 8 Summary of Previous Studies

Study/Population	N^*	Sources of Uncertainty	Judged Probabilities $(A, S-A)$	Judged Probabilities $(A_1, \ldots, A_n)$	Certainty Equivalents ΣC	Certainty Equivalents $\%V$
a. NBA Fans	27	Playoff Game	0.99	1.40	1.40	93
		SF Temperature	0.98	1.47	1.27	77
b. NFL Fans	40	Super Bowl	1.01	1.48	1.31	78
		Dow Jones	0.99	1.25	1.16	65
c. Stanford Students	45	SF Temperature	1.03	2.16	1.98	88
		Beijing Temperature	1.01	1.88	1.75	82
d. Options Traders (San Francisco)	32	Microsoft	1.00	1.40	1.53	89
		General Electric	0.96	1.43	1.50	89
e. Options Traders (Chicago)	28	IBM	1.00	1.27	1.47	82
		Gannett Co.	0.99	1.20	1.13	64
		Median	1.00	1.42	1.44	82

The first three columns identify the subject population, sample sizes, and sources of uncertainty. Studies *a*, *b*, and *c* are reported in Tversky and Fox (1995) and are based on a sixfold partition. Studies *d*, and *e* are reported in Fox et al. (1996), and are based on a fourfold partition. The next two columns present the median sum of judged probabilities for a binary partition $(A, S-A)$ and for n-fold partitions $(A_1, \ldots, A_n)$ of S. The next column, labeled ΣC, presents the median sum of normalized certainty equivalents over an n-fold partition of S. The final column, labelled $\%V$, presents the percentage of subjects who violated the partition inequality. A few table entries are based on smaller samples than indicated because of missing data.

principles advanced in support theory and prospect theory. Alternatively, one might be tempted to account for these findings by a bias toward the midpoint of the response scale. This bias could be induced by anchoring on the midpoint of the scale, or by a symmetric error component that is bounded by the endpoints of the response scale. Although such response bias may contribute to subadditivity in some studies, it cannot provide a satisfactory account of this phenomenon. First, there is compelling evidence for bounded subadditivity in simple choices between uncertain prospects (see e.g., Tversky and Kahneman 1992, tables 1 and 2; Wu and Gonzalez 1996) that cannot be explained as a response bias.[16] Second, response bias cannot account for the observation that unpacking the description of a target event can increase the attractiveness of the corresponding prospect, nor can it account for the resulting nonmonotonicities described above. Third, a symmetric bias toward the midpoint of the response scale cannot explain the observation that both cash equivalents and decision weights for complementary prospects generally sum to less than one. Finally, it should be noted that the significance of subadditivity to the prediction of judgment and choice is not affected by whether it is interpreted as a feature of the evaluation process, as a response bias, or as a combination of the two.

[16] The studies of Wu and Gonzalez (1996) provide evidence of concavity for low probabilities and convexity for moderate to high probabilities, which are stronger than lower and upper subadditivity, respectively.

Source Preference

There is evidence that people's willingness to bet on an uncertain event depends not only on the degree of uncertainty but also on its source. We next review this phenomenon and discuss its relation to the belief-based account.

A person exhibits source preference if he or she prefers to bet on a proposition drawn from one source rather than on a proposition drawn from another source, and also prefers to bet against the first proposition rather than against the second. Source preference was first illustrated by Ellsberg (1961) using the following example. Consider an urn containing 50 red and 50 black balls, and a second urn containing 100 red and black balls in an unknown proportion. Suppose you are offered a cash prize if you correctly guess the color of a ball drawn blindly from one of the urns. Ellsberg argued that most people would rather bet on a red ball from the first urn than on a red ball from the second, and they also would rather bet on a black ball from the first urn than on a black ball from the second. This pattern has been observed in several studies (see Camerer and Weber 1992 for a review). The preference to bet on clear or known probabilities rather than vague or unknown probabilities has been called *ambiguity aversion*.

More recent research has shown that although people exhibit ambiguity aversion in situations of complete ignorance (e.g., Ellsberg's urn), they often prefer betting on their vague beliefs than on matched chance events. Indeed, the evidence is consistent with a more general account, called the *competence hypothesis*: people prefer to bet on their vague beliefs in situations in which they feel particularly competent or knowledgeable, and they prefer to bet on chance when they do not (Heath and Tversky 1991). For example, subjects who were knowledgeable about football but not about politics preferred to bet on the outcome of professional football games than on matched chance events, but they preferred to bet on chance than on the results of a national election. Analogously, subjects who were knowledgeable about politics but not about football preferred to bet on the results of an election than on matched chance events, but they preferred to bet on chance than on football.[17]

The present studies provide some evidence for source preference that is consistent with the competence hypothesis. Recall that subjects in Study 1 were recruited for their interest in professional basketball. Indeed, these subjects preferred betting on basketball to betting on matched chance events: the median certainty equivalent for the Eastern Conference ($79) and the Western Conference ($74) were both greater than the median certainty equivalent for the 50-percent chance prospect ($69). In contrast, subjects in Study 2 did not have special expertise regarding the simulated economy. Indeed, these subjects generally preferred betting on chance to betting on the economic indicators. For example, the median certainty equivalent for both inflation and interest going up ($690) was the same as the median certainty equivalent for the 50-percent chance prospect, but the median certainty equivalent for the complementary event ($470) was considerably lower.

It is evident that source preference cannot be explained by the present model, though it can be accommodated by a more general belief-based account. For example, we can generalize equation (1) by letting $W(A) = F[P(A)]$ so that the transformation F of judged probability depends on the source of uncertainty. One convenient parameterization may be defined by $W(A) = (w[P(A)])^{\theta}$, where $\theta > 0$ is inversely related to the attractiveness of the source.[18] These generalizations no longer satisfy Equation (5), but they maintain the decomposition of W into two components: P, which reflects a person's belief in the likelihood of the target event; and F (or w^{θ}), which reflects a person's preference to bet on that belief.

Practical Implications

The two-stage model may have important implications for the management sciences and related fields. First, the unpacking principle implies that the particular descriptions of events on which outcomes depend may affect a person's willingness to act. Hence, the attractiveness of an opportunity such an investment might be increased by unpacking the ways in which the investment could be profitable; willingness to take protective action such as the purchase of insurance might be increased by unpacking the ways in which a relevant mishap might occur.

Second, violations of the partition inequality suggest that people are willing to pay more for a prospect when components are evaluated separately; thus,

[17] To complicate matters further, Fox and Tversky (1995) have shown that ambiguity aversion, which has been commonly observed when people evaluate both clear and vague propositions jointly, seems to diminish or disappear when people evaluate only one of these propositions in isolation.

[18] Alternatively, one might accommodate source preference by varying a parameter of the risky weighting function that increases or decreases weights throughout the unit interval. For example, one can vary β of Prelec's (1998) two-parameter risky weighting function, $w(p) = \exp(-\beta(-\ln p)^{\alpha})$, where $\beta > 0$ is inversely related to the attractiveness of the source. This has the advantage of manipulating the "elevation" of the function somewhat independently of its degree of "curvature." For more on elevation and curvature of the weighting function, see Gonzalez and Wu (1998).

they are willing to pay a premium, on average, for specificity. When such decisions are aggregated over time or across individuals within an organization, this pattern can lead to certain losses. To illustrate, the first author ran a classroom exercise in which MBA students were divided into six "firms" of eight students each, and each student was asked to decide their firm's maximum willingness to pay for an "investment" that would yield $100,000 depending on future movement of indicators in the U.S. economy. The state space was partitioned into eight events (one for each student) so that each firm's portfolio of investments resulted in a certain return of exactly $100,000. Nevertheless, the six firms reported willingness-to-pay for the eight investments that summed to between $107,000 and $210,000.

5. Concluding Remarks

We have provided evidence that decision weights under uncertainty can be predicted from judged probabilities of events and risky decision weights. To the extent that the two-stage model reflects the psychological process underlying decision under uncertainty, this model suggests two independent sources of departure from the classical theory: a belief-based source (subadditivity of judged probability) and a preference-based source (nonlinear weighting of chance events). While the development of effective prescriptions for correcting such bias awaits future investigation, this decomposition of the weighting function offers a new approach to the modeling of decision under uncertainty that integrates probability judgment into the analysis of choice.[19]

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