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# Options Traders Exhibit Subadditive Decision Weights

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## *Abstract*

Professional options traders priced risky prospects as well as uncertain prospects whose outcomes depended on future values of various stocks. The prices of the risky prospects coincided with their expected value, but the prices of the uncertain prospects violated expected utility theory. An event had greater impact on prices when it turned an impossibility into a possibility or a possibility into a certainty than when it merely made a possibility more or less likely, as predicted by prospect theory. This phenomenon is attributed to the subadditivity of judged probabilities.

**Key words:** risk, uncertainty, decision weights, subadditivity

In recent years, several authors have attempted to accommodate violations of expected utility theory by replacing additive probabilities with nonadditive decision weights (e.g., Kahneman and Tversky, 1979; Quiggin, 1982; Yaari, 1987; Gilboa, 1987; Schmeidler, 1989; Wakker, 1989; Luce and Fishburn, 1991; Tversky and Kahneman, 1992). Empirical evidence suggests that decision weights follow a distinctive pattern: an event has greater impact when it turns impossibility into possibility or possibility into certainty than when it merely makes a possibility more or less likely. This principle, called *bounded subadditivity* (Tversky and Wakker, 1995), can explain the certainty effect, the possibility effect, and other violations of expected utility theory (see Wu and Gonzalez, 1996). Tversky and Fox (1995) tested this principle in a series of studies involving risky prospects as well as uncertain prospects whose outcomes depended on upcoming sporting events, future temperatures, and future values of the Dow–Jones index. Bounded subadditivity was observed for both risk and uncertainty, and this phenomenon was more pronounced for uncertainty than for risk.

The participants in these studies were Stanford students recruited specifically for their sports acumen; some were actually paid on the basis of their responses. The question arises whether expert decision makers who evaluate uncertain prospects as part of their daily professional activity also exhibit bounded subadditivity in their domain of expertise. To answer this question, we recruited professional options traders and support staff on the floors of the Pacific Stock Exchange and Chicago Board Options Exchange, and asked them to price both risky prospects and uncertain prospects based on future values of various stocks. Options traders can be considered experts in decision making under uncertainty. They are schooled in the calculus of chance; they have a great deal of experience assessing uncertainty; they are trained to identify and exploit arbitrage opportunities; and they are selected for their ability to do so. Consequently, options traders—perhaps more than other groups—could be expected to avoid the biases commonly observed in studies of decision under uncertainty.

## 1. Theory

We first introduce the theoretical framework of cumulative prospect theory that underlies the analysis presented in this article (Tversky and Kahneman, 1992; Wakker and Tversky, 1993). Because we consider here only prospects with nonnegative outcomes, this representation is essentially equivalent to the rank-dependent utility model. Let $S$ be a set whose elements are interpreted as states of the world. Subsets of $S$ are called *events*. Thus, $S$ corresponds to the certain event, and $\phi$ is the null event. A weighting function $W$ (on $S$) is a mapping that assigns to each event in $S$ a number between 0 and 1 such that $W(\phi) = 0$, $W(S) = 1$, and $W(A) \geq W(B)$ if $A \supset B$. Such a function is also called a *capacity*, or a nonadditive probability.

Let $(x_1, A_1;\ldots;x_n, A_n)$ be a prospect that offers $\$x_i$ if event $A_i$ obtains. Assume that $(A_1,\ldots,A_n)$ is a partition of $S$ and that $0 \leq x_1 \leq\ldots\leq x_n$. The value of such a prospect is given by

$$\sum_{i=1}^{n} v(x_i)\pi_i,$$

$$\text{where } \pi_n = W(A_n) \text{ and } \pi_i = W(A_i \cup\ldots\cup A_n) - W(A_{i+1} \cup\ldots\cup A_n),$$
$$i = 1,\ldots,n-1.$$

Thus, the value of a nonnegative prospect is determined jointly by the value function for monetary gains $v$, and the weighting function $W$, defined on $S$. We assume that $v$ is strictly increasing and that $v(0) = 0$. Note that this form reduces to an expected utility model if $W$ is additive, that is, if $W(A \cup B) = W(A) + W(B)$, for $A \cap B = \phi$. Prospect theory, however, imposes the following weaker constraints.

**(1) Lower subadditivity:** $W(A) \geq W(A \cup B) - W(B)$, provided $A$ and $B$ are disjoint and $W(A \cup B)$ is bounded away from one.[1] This inequality captures the possibility effect: the impact of an event $A$ is greater when it is added to the null event than when it is added to some nonnull event $B$.

**(2) Upper subadditivity:** $W(S) - W(S - A) \geq W(A \cup B) - W(B)$, provided $A$ and $B$ are disjoint and $W(B)$ is bounded away from zero. This inequality captures the certainty effect: the impact of an event $A$ is greater when it is subtracted from the certain event $S$ than when it is subtracted from some uncertain event $A \cup B$. If we define the dual function, $W'(A) = 1 - W(S - A)$, then upper subadditivity can be expressed as $W'(A) \geq W'(A \cup B) - W'(B)$. Thus, upper subadditivity of $W$ is equivalent to lower subadditivity of $W'$.

A weighting function $W$ satisfies *bounded subadditivity*, or subadditivity (SA) for short, if it satisfies both (1) and (2) above. This principle extends to uncertainty the notion that increasing the probability of winning a prize from 0 to $p$ has more impact than increasing the probability of winning from $q$ to $q + p$, provided $q + p < 1$, and decreasing the probability of winning from 1 to $1\text{-}p$ has more impact than decreasing the probability of winning from $q + p$ to $q$, provided $q > 0$. Note that risk can be viewed as a special case of uncertainty where probability is defined via a standard chance device so that the probabilities of outcomes are known. In the following studies, we estimate decision weights of options traders for both risky and uncertain prospects and test the principle of bounded subadditivity.

## 2. *Experiments*

In this section, we describe two studies. The first study involves pricing and matching risky prospects. The second study involves pricing uncertain prospects and assessing the probabilities of uncertain events.

### 2.1. *Study 1: Decision under risk*

**Subjects.** We recruited 88 participants, consisting of options traders ($N = 55$) and support staff ($N = 33$) on the options floor of the Pacific Stock Exchange in San Francisco. Because we found no significant differences between these groups, their data were combined. The respondents were 81 men and 7 women. Median age of respondents was 30, and median experience working on the floor of the Pacific exchange was about five years.

**Procedure.** The subjects were approached individually by the experimenter on the floor of the exchange, and asked to participate in a brief study of decision making by completing a brief questionnaire. They were offered a \$1 California lottery ticket and a chance to win up to \$150 on the basis of their responses. To determine who was to play for real money, subjects chose a number from 1 to 20 and later rolled a 20-sided die; those who accurately predicted the roll of the die were selected to play for real. The participants were first asked to price risky prospects with a single positive outcome, and were then asked to match risky prospects with two positive outcomes. Because the analysis of the pricing data is based on the results of the matching data, the latter task will be described first. An incentive-compatible payoff scheme, based on Becker, De Groot, and Marschak (1964), was employed. Six subjects were selected at random to play for real money and they were paid, on average, $68 each.

**Task 1 (matching).** Two problems, each involving a complete and an incomplete prospect, were used to assess the value function. All prospects would pay a high prize if a fair die would land 1, a lower prize if the die would land 2, and nothing otherwise. In the first problem, the complete prospect offered $100 if the die would land 1, $50 if it would land 2 (and nothing otherwise). The incomplete prospect offered $$x$ if the die would land 1, $25 if it would land 2 (and nothing otherwise). Subjects were asked to report the value of $x$ for which they found the two prospects equally attractive. The second problem was similar to the first: the complete prospect offered $90 if the die would land 1 and $60 if it would land 2; the incomplete prospect offered $$y$ if the die would land 1 and $35 if it would land 2. The order of presentation of the two problems was counterbalanced.

The median value of $x$ reported by subjects was $125, and the median value of $y$ was $115, in accord with a linear value function. Cumulative prospect theory (as well as expected utility theory) implies that the observed difference between the high outcomes of the matched prospects equals the difference between their low outcomes if and only if the value function is linear. This condition was satisfied by 75% and 80% of subjects in our first and second problems, respectively. The present finding contrasts with the results of similar studies using other populations that have yielded concave value functions for gains over this range.

**Task 2 (pricing).** This task was designed to estimate decision weights for risky prospects. Subjects were asked to state their minimum selling prices for nine prospects, each offering to pay $150 with a specified probability. The probability of winning varied from .10 to .90, in increments of .10. Half the subjects received the prospects in an order of increasing probability; half the subject received the prospects in an order of decreasing probability.

According to prospect theory, the value of a risky prospect $(x,p)$ paying $$x$ with probability $p$, and $0 otherwise is $v(x)w(p)$, where $w$ is the risky weighting function. Let $y_p$ be the minimum selling price of the prospect ($150,$p$); hence $v(y_p) = v(\$150)w(p)$. The results of the matching task indicate that $v$ is linear; hence the decision weight $w(p)$ can be estimated by the ratio $y_p/150$.

The circles in figure 1 depict the median value of $w$, across subjects, for each of the nine values of $p$. The data yield a perfectly linear weighting function. This result contrasts sharply with the typical $S$-shaped weighting function generally observed in studies of decision under risk (see, e.g., Camerer, 1995), depicted by triangles in figure 1. The latter weights were obtained by Tversky and Kahneman (1992) using a comparable set of prospects in a study of Stanford and U.C. Berkeley students. Taken together, the results of Study 1 indicate that options traders, unlike most other subjects, price risky prospects by their expected actuarial value.

It could be argued that this observation is the result of a familiar calculation rather than genuine risk-neutrality in this outcome range. While familiarity with expected value might

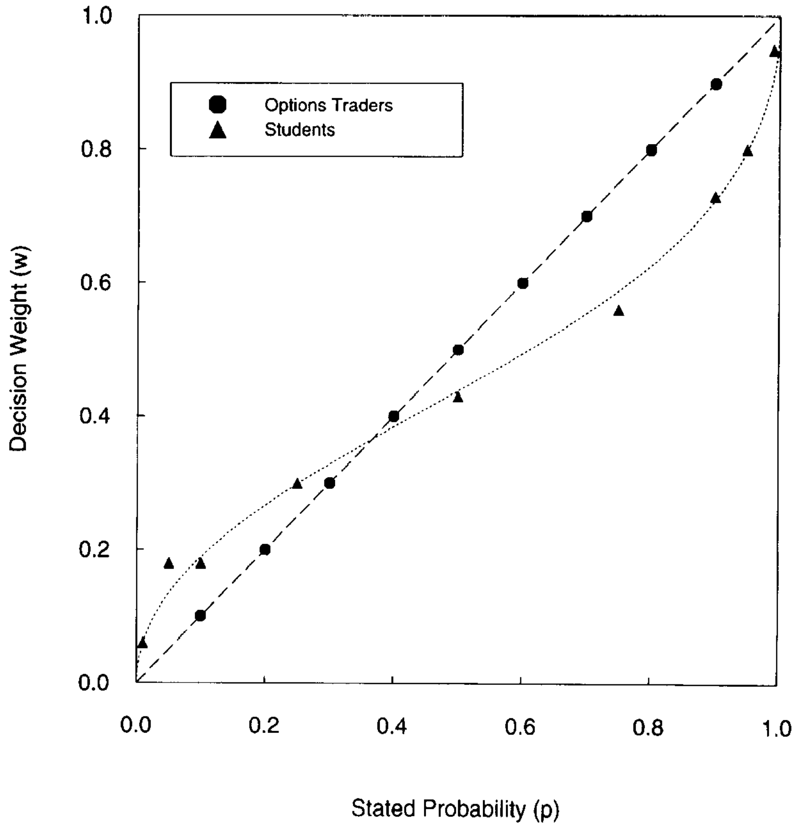



*Figure 1.* Decision weights of options traders and of students for chance prospects.

have facilitated these responses, it should be recalled that the traders knew that they might be playing for real money. Some of them actually won as much as $150. Had their true preferences been significantly at odds with expected value, it is doubtful that they would have comformed to this rule.

### *2.2. Study 2: Decision under uncertainty*

**Subjects.** Two groups participated in this study. The first group ($N$ = 32) was recruited from the options floor of the Pacific Stock Exchange (PSE) and included options traders ($N$ = 15) and support staff ($N$ = 17). Median experience at the Pacific exchange was five years. The second group ($N$ = 28) was recruited from the floor of the Chicago Board Options Exchange (CBOE) and included options traders ($N$ = 19) and support staff ($N$ = 9). Median experience at the Chicago exchange was nearly four years.

**Procedure.** Subjects were approached on the trading floor and asked to participate in a study of decision making. In exchange for completing the survey, subjects were promised a $1 California lottery ticket (PSE subjects) or Illinois lottery ticket (CBOE subjects) and a chance to win up to $150 on the basis of their responses. An incentive-compatible payoff scheme, based on Becker, De Groot, and Marschak (1964) was employed. To determine who was to play for real money, subjects in San Francisco chose a number from 1 to 20 and rolled a 20-sided die; those who accurately predicted the roll of the die were selected to play for real. In Chicago, three surveys were chosen at random from those returned. Using this procedure, four subjects were selected to play for real money and received an average of $85 each.

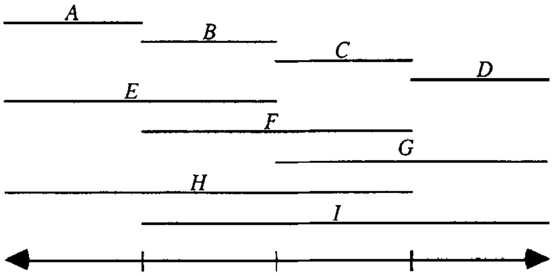
**Design.** Participants were asked to report their minimum selling price for prospects of the form ($150,$A$) that offered $150 if a target event $A$ would occur and nothing otherwise. Target events were defined by the closing price of a particular stock two weeks in the future. Two stocks were selected for each group: a familiar stock whose options traded on subjects' local exchange and a less familiar stock whose options traded on a different exchange. In each case, the stocks were selected to have comparable closing prices and price volatilities. We provided subjects with the previous day's closing price of each stock, and current price and volatility information was readily available on the exchange floors. For the PSE subjects, the more familiar stock was Microsoft (MSFT), and the less familiar stock was General Electric (GE). For the CBOE subjects, the more familiar stock was IBM, and the less familiar stock was Gannett Company Incorporated (GCI). Subjects' ratings of these stocks on a ten-point scale of familiarity confirmed our a priori classification. For the PSE subjects, median familiarity was 7 for MSFT and 3 for GE; for the CBOE subjects, median familiarity was 7 for IBM and 1 for GCI.

For each stock we selected nine target events, defined by the closing price of that stock two weeks in the future. These events are represented by horizontal lines in figure 2. For example, event $B$ for Microsoft is "the closing price of Microsoft is equal to or greater than $88 and less than $94 per share." Thus, it is defined by the inequality $88 \leq$ MSFT $< 94$. The design depicted in figure 2 provides seven tests of lower SA and three tests of upper SA, listed in the lower part of the figure. For example, the first test of lower SA, $W(A) \geq W(E) - W(B)$, follows from the relation $A \cup B = E$.

Four additional prospects based on the future value of two different stocks were also interspersed among the target prospects in each survey to make the relationship among the target events less transparent. The order in which prospects were presented was randomized individually for each participant, constrained so that prospects based on a particular stock did not appear consecutively.

Following the pricing of prospects, subjects judged the probabilities of all target events. The order of presentation was again randomized for each subject.

**Results.** As in Study 1, there were no significant differences between the traders and support staff, so their data were combined. The pricing data of five subjects from the PSE group were excluded, because these subjects did not follow the instructions properly.




| | | | |
|---|---|---|---|
| MSFT | 88 | 94 | 97 |
| GE | 92 | 98 | 101 |
| IBM | 57 | 61 | 64 |
| GCI | 49 | 53 | 56 |

| Lower SA | Upper SA |
|---|---|
| $W(A) \geq W(E) - W(B)$ | $1 - W(I) \geq W(E) - W(B)$ |
| $W(B) \geq W(F) - W(C)$ | $1 - W(I) \geq W(H) - W(F)$ |
| $W(C) \geq W(G) - W(D)$ | $1 - W(G) \geq W(H) - W(C)$ |
| $W(E) \geq W(H) - W(C)$ | |
| $W(A) \geq W(H) - W(F)$ | |
| $W(F) \geq W(I) - W(D)$ | |
| $W(B) \geq W(I) - W(G)$ | |

*Figure 2.* Schematic representation of events on which bets were based, and tests of lower subadditivity and upper subadditivity resulting from this design.

**Decision weights.** Let $y_A$ be the minimum selling price of the prospect ($150,$A$). Thus, $v(y_A)$ = v($150)$W(A)$, and $W(A) = v(y_A)/v(\$150)$. As shown in Study 1, the value function for gains for this population was essentially linear in the range of $0 to $150. Hence, the decision weight $W(A)$ is given by the ratio $y_A/150$. Using these estimates, we computed for each subject the mean value of $W(A) + W(B)$ and the mean value of $W(A \cup B)$ across all tests of lower SA for the more familiar stock on each exchange. Figure 3a plots the latter mean against the former mean, separately for each subject. If $W$ were additive (i.e., $W(A) + W(B) = W(A \cup B)$), as implied by expected utility theory, the points should lie on the identity line. In contrast, the points for 46 of the 55 subjects lie below the identity line, in accord with lower SA ($p < .001$, by sign test). Similarly, figure 3b plots separately for each subject the mean value of $W'(A) + W'(B)$ against the mean value $W'(A \cup B)$, across all tests of upper SA. Again, the points for 45 of the 55 subjects lie below the identity line, in accord with upper SA ($p < .001$, by sign test). Similar results were obtained for the less familiar stocks.

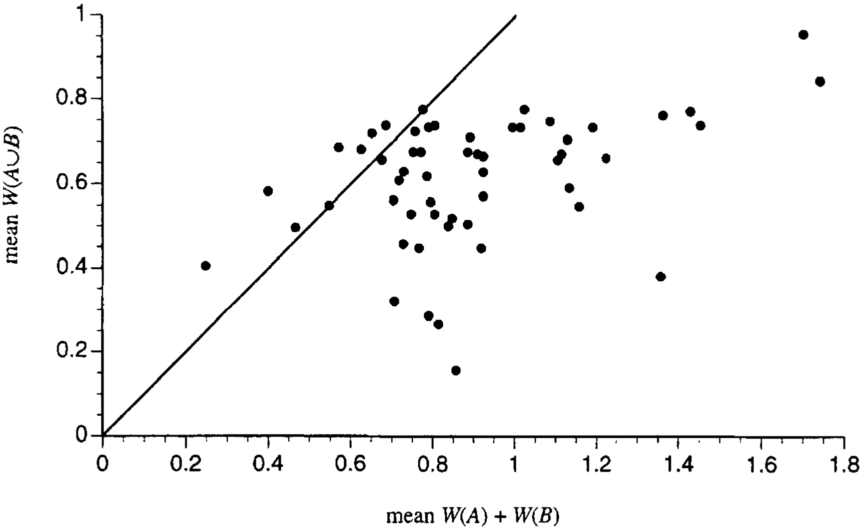



*Figure 3a.* Lower subadditivity of decision weights for options traders, plotting the mean value of $W(A \cup B)$ against the mean value of $W(A) + W(B)$ for each subject.

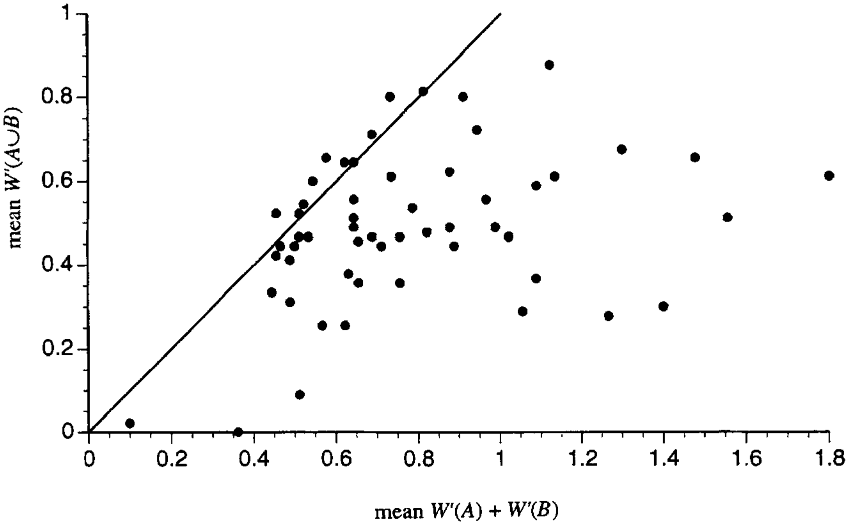



*Figure 3b.* Upper subadditivity of decision weights for options traders, plotting the mean value of $W'(A \cup B)$ against the mean value of $W'(A) + W'(B)$ for each subject. Recall that $W'(A) = 1 - W(S - A)$.

To obtain quantitative estimates of the degree of lower SA and upper SA, respectively, define for each subject

$$D(A,B) \equiv W(A) + W(B) - W(A \cup B),$$

$$D'(A,B) \equiv W'(A) + W'(B) - W'(A \cup B).$$

Let $d$ and $d'$ be the mean values of $D$ and $D'$, respectively, for a given subject. Under expected utility theory, $d = d' = 0$, whereas lower and upper SA imply $d \geq 0$ and $d' \geq 0$. It is worth noting that the values of $d$ and $d'$ for each subject are proportional to the distance between the point representing that subject and the identity line in figures 3a and 3b, respectively. The left-hand part of table 1 presents the values of $d$ and $d'$, based on the median selling price of each prospect across subjects. As predicted, all entries in the left-hand part of table 1 are substantially greater than zero; their average is .19 for $d$, and .21 for $d'$. Thus, the mean sum of the decision weights of two disjoint events (.84) exceeds the mean decision weight of their union (.65) by .19, or about 29%. The results for the dual function are similar.

**Judged probability.** We distinguish between decision weights, derived from preferences, and degree of belief, expressed by direct judgments of probability. To investigate the relationship between these constructs, we have applied the preceding analysis of decision weights to judged probabilities.

Let $P(A)$ denote the judged probability of uncertain event $A$, and let $P'(A) = 1 - P(S - A)$ denote its dual. Figure 4a presents the mean value for each subject of $P(A) + P(B)$ against the mean value of $P(A \cup B)$ for the more familiar stock, and figure 4b presents the mean value for each subject of $P'(A) + P'(B)$ against the mean value of $P'(A \cup B)$ for the more familiar stock. Thus figures 4a and 4b that describe judged probability are the counterparts of figures 3a and 3b, which describe decision weights. If judged probabilities were additive, in accord with probability theory, the points should lie on the identity line. In contrast, the data reveal pervasive subadditivity: 55 of 60 points lie below the identity line in figure 4a and 55 of 60 points lie below the identity line in figure 4b ($p < .001$, by sign test). Similar results were obtained for the less familiar stocks.

Define $D$, $D'$, $d$, and $d'$ as above, using the judged probability $P(A)$ instead of the decision weight $W(A)$. The right-hand side of table 1 presents the values of $d$ and $d'$, based on the median judged probabilities across subjects. As was the case for decision weights,

*Table 1.* Values of $d$ and $d'$ that measure lower and upper SA, respectively, for decision weights and judged probability, based on median responses across subjects.

| | Decision weight ($W$) | | Judged probability ($P$) | |
|---|---|---|---|---|
| stock | d | d′ | d | d′ |
| MSFT | .21 | .24 | .14 | .17 |
| GE | .20 | .22 | .22 | .26 |
| IBM | .21 | .21 | .12 | .14 |
| GCI | .12 | .18 | .14 | .17 |

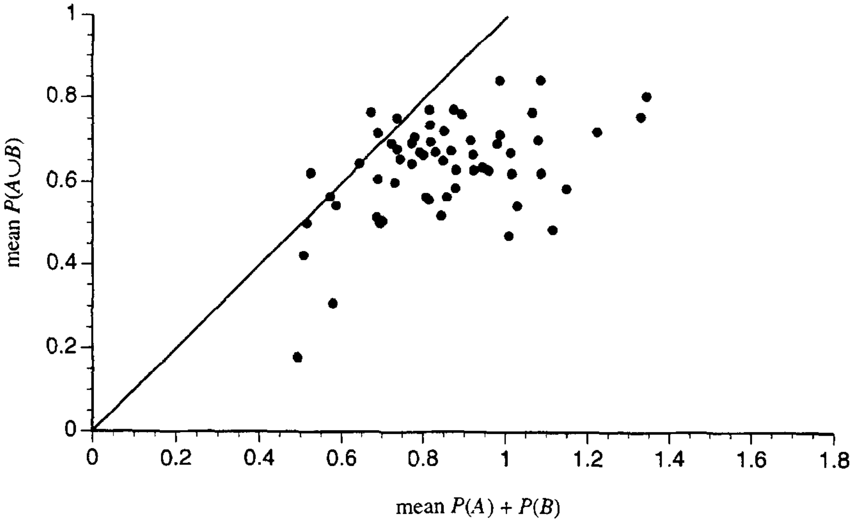



*Figure 4a.* Lower subadditivity of judged probability for options traders, plotting the mean value of $P(A \cup B)$ against the mean value of $P(A) + P(B)$ for each subject.

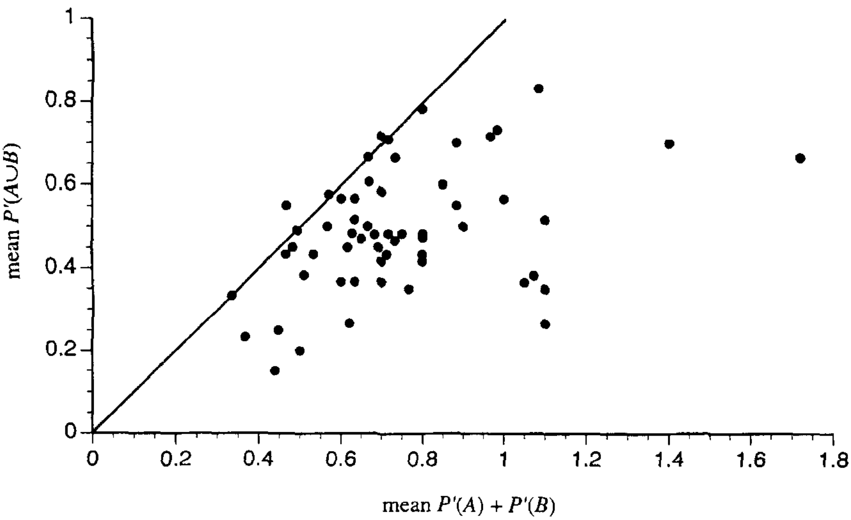



*Figure 4b.* Upper subadditivity of judged probability for options traders, plotting the mean value of $P'(A \cup B)$ against the mean value of $P'(A) + P'(B)$ for each subject.

all values of $d$ and $d'$ are greater than zero; their average is .16 for $d$, and .19 for $d'$. Thus, the mean sum of the judged probabilities of two disjoint events (.84) exceeds the mean judged probability of their union (.68) by .16 or about 24%. The results for the dual function are similar. These findings are consistent with the analysis of judged probability advanced in support theory (Tversky and Koehler, 1994.)[2]

Comparison of the left-hand and right-hand parts of table 1 reveals a comparable degree of subadditivity for judged probability (mean = .17) and of decision weights (mean = .20). These data are consistent with a two-stage process in which decision makers first assesses the probability $P$ of an uncertain event $A$, then transform this value according to their attitudes toward risk (cf. Tversky and Fox, 1995). Because our options traders exhibited risk-neutrality, their uncertain decision weights $W(A)$ should roughly correspond to their judged probabilities $P(A)$. Figure 5 plots median decision weights against median judged probabilities for each of the nine target events, labeled separately for each of the

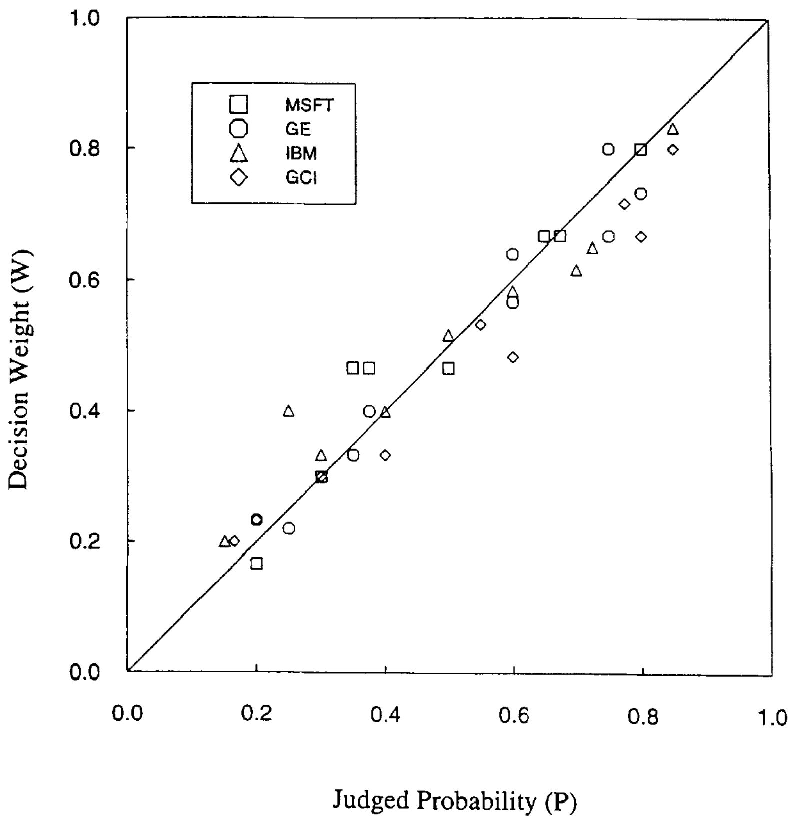



*Figure 5.* Median decision weights for options traders plotted against median judged probability, labeled separately for each stock.

four stocks. As predicted, the points are scattered along the identity line, suggesting that the observed subadditivity of decision weights is attributable to the subadditivity of judged probability.

## 3. Discussion

The main finding of the present studies is a remarkable contrast between options traders' behavior under risk and uncertainty. For risky prospects, where objective probabilities were known, traders maximized expected value. However, for uncertain prospects, where a subjective assessment of uncertainty is required, these respondents consistently violated expected utility theory by exhibiting pronounced subadditivity.

Experimental studies of decision making under uncertainty have sometimes been criticized on the grounds that the subjects (mostly college students) were required to make decisions involving events with which they did not have much prior experience. The present findings indicate that professional options traders exhibited marked bounded subadditivity in decisions involving events about which they have a great deal of knowledge with which they have a great deal of experience. Microsoft and IBM options are among the most actively traded issues on their respective exchanges. Evidently, years of experience in forecasting price movements of Microsoft or IBM and making trades on the basis of these beliefs were not sufficient to expunge subadditivity. The question of whether this bias can produce inefficiency in trading awaits future investigation.

## Acknowledgment

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## Notes

1. The boundary conditions are needed to ensure that we always compare an interval that includes an endpoint to an interval that is bounded away from the other endpoint (see Tversky and Wakker, 1995, for a more rigorous formulation).
2. Support theory also implies that the judged probabilities of an event and its complement sum to one. Indeed, the median value, across subjects, of these sums is 1.00, 0.96, 1.00, 0.99, for MSFT, GE, IBM, and GCI, respectively.

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